GCSE Maths – Number

Exact Values and Surds

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of exact value questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Calculate $\frac{4}{7} + \frac{2}{3}$

Step 1: Manipulate each fraction so that they have the same denominator (bottom number).

To do this, we need to find the lowest common multiple of both denominators. Here, it is 21. To change a fraction, we need to multiply the numerator and denominator by the same number.

$$\frac{4}{7} \times \frac{3}{3} = \frac{12}{21}$$

$$\frac{2}{3} \times \frac{7}{7} = \frac{14}{21}$$

Step 2: Perform the operation on the fractions that have the same denominator.

$$\frac{12}{21} + \frac{14}{21} = \frac{26}{21}$$

Step 3: Simplify the fraction if possible and leave in its exact form.

This fraction cannot be simplified, so we leave it as

$$\mathbf{\frac{26}{21}}$$

Guided Example

Calculate $\frac{10}{12} - \frac{3}{5}$

Step 1: Manipulate each fraction so that they have the same denominator (bottom number).

$\frac{10 \times 5}{12 \times 5} - \frac{3 \times 12}{5 \times 12} = \frac{50}{60} - \frac{36}{60}$

Step 2: Perform the operation on the fractions that have the same denominator.

Subtract numerators: $\frac{50}{60} - \frac{36}{60} = \frac{14}{60}$

Step 3: Simplify the fraction if possible and leave in its exact form.

$\frac{14}{60} = \frac{7 \times 2}{30 \times 2} = \frac{7}{30}$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

1. Calculate the following, leaving your answer in exact form.

a) $\frac{5}{9} + \frac{1}{3}$

$\frac{5}{9} + \frac{1}{3} = \frac{5\times3}{9\times3} + \frac{1\times9}{3\times9} = \frac{15}{27} + \frac{9}{27} = \frac{24}{27} = \frac{8\times3}{9\times3} = \frac{8}{9}$

Cross multiply the denominators

Cancel out the common Factor (3).

b) $1\frac{1}{2} - \frac{4}{7}$

$1\frac{1}{2} - \frac{4}{7} = \frac{3}{2} - \frac{4}{7} = \frac{3\times7}{2\times7} - \frac{4\times2}{7\times2} = \frac{21}{14} - \frac{8}{14} = \frac{13}{14}$

Convert mixed number to an improper fraction

Cross multiply the denominators

c) $3\frac{2}{3} + 2\frac{3}{4}$

$3\frac{2}{3} + 2\frac{3}{4} = \frac{11}{3} + \frac{11}{4} = \frac{11\times4}{3\times4} + \frac{11\times3}{4\times3} = \frac{44}{12} + \frac{33}{12} = \frac{77}{12}$

Convert mixed numbers to improper fractions

Cross multiply the denominators

d) $\frac{3}{8} - \frac{7}{10}$

$\frac{3}{8} - \frac{7}{10} = \frac{3\times10}{8\times10} - \frac{7\times8}{10\times8} = \frac{30}{80} - \frac{56}{80} = -\frac{26}{80} = -\frac{13\times2}{40\times2} = -\frac{13}{40}$

Section B

Worked Example

Calculate $\frac{3}{7} \times \frac{1}{2}$

Step 1: To multiply exact fractions, multiply the numerators of each, and multiply the denominators of each.

$$\frac{3}{7} \times \frac{1}{2} = \frac{3 \times 1}{7 \times 2} = \frac{3}{14}$$

Step 2: Write the final fraction in exact form and check if it can be simplified.

The final answer cannot be simplified, so we leave the answer as

$$\frac{3}{14}$$

Guided Example

Calculate $\frac{3}{4} \times \frac{6}{7}$

Step 1: To multiply exact fractions, multiply the numerators of each, and multiply the denominators of each.

$\frac{3}{4} \times \frac{6}{7} = \frac{3 \times 6}{4 \times 7} = \frac{18}{28}$

Step 2: Write the final fraction in exact form and check if it can be simplified.

The final answer can be simplified by finding common factors:

$\frac{18}{28} = \frac{9 \times 2}{14 \times 2} = \frac{9}{14}$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

2. Calculate the following, leaving your answer in exact form.

a) $\frac{5}{6} \times \frac{2}{3}$

$\frac{5}{6} \times \frac{2}{3} = \frac{5 \times 2}{6 \times 3} = \frac{10}{18} = \frac{5 \times 2}{9 \times 2} = \frac{5}{9}$

Multiply the numerators and denominators together

Simplify by Finding common factors.

b) $2\frac{1}{4} \times \frac{7}{9}$

$2\frac{1}{4} \times \frac{7}{9} = \frac{9}{4} \times \frac{7}{9} = \frac{9 \times 7}{4 \times 9} = \frac{63}{36} = \frac{7 \times 9}{4 \times 9} = \frac{7}{4}$

Convert mixed number to an improper fraction

Multiply the numerators and denominators together

Simplify by Finding common factors.

c) $\frac{-4}{5} \times 1\frac{2}{7}$

$-\frac{4}{5} \times 1\frac{2}{7} = \frac{-4}{5} \times \frac{9}{7} = \frac{-4 \times 9}{5 \times 7} = \frac{-36}{35}$

Convert mixed number to an improper fraction

Multiply the numerators and denominators together

d) $4\frac{2}{3} \times \frac{2}{3}$

$4\frac{2}{3} \times \frac{2}{3} = \frac{14}{3} \times \frac{2}{3} = \frac{14 \times 2}{3 \times 3} = \frac{28}{9}$

Convert mixed number to an improper fraction

Multiply the numerators and denominators together

Section C

Worked Example

Calculate $\frac{3}{7} \div \frac{1}{6}$

Step 1: To divide exact fractions, we need to flip the numerator and denominator of the second fraction.

Flipping the second fraction gives us $\frac{6}{1}$.

Step 2: We then change the sign from division to multiplication and multiply the fractions (multiply the numerators and denominators).

$$\frac{3}{7} \times \frac{6}{1} = \frac{18}{7}$$

Step 3: Simplify the fraction if possible and leave in its exact form.

The final answer cannot be simplified, so we leave it as:

$$\frac{18}{7}$$

Guided Example

Calculate $\frac{6}{7} \div \frac{5}{3}$

Step 1: To divide exact fractions, we need to flip the numerator and denominator of the second fraction.

$\frac{5}{3}$ becomes $\frac{3}{5}$

Step 2: We then change the sign from division to multiplication and multiply the fractions (multiply the numerators and denominators).

$$\frac{6}{7} \div \frac{5}{3} = \frac{6}{7} \times \frac{3}{5} = \frac{6 \times 3}{7 \times 5} = \frac{18}{35}$$

Step 3: Simplify the fraction if possible and leave in its exact form.

Fraction cannot be simplified: $\frac{18}{35}$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

3. Calculate the following, leaving your answer in exact form.

a) $\frac{2}{3} \div \frac{1}{7}$

$\frac{2}{3} \div \frac{1}{7} = \frac{2}{3} \times \frac{7}{1} = \frac{2 \times 7}{3 \times 1} = \frac{14}{3}$

Flip the second fraction and multiply.

Multiply the numerators and denominators together

b) $\frac{5}{6} \div \frac{4}{3}$

$\frac{5}{6} \div \frac{4}{3} = \frac{5}{6} \times \frac{3}{4} = \frac{5 \times 3}{6 \times 4} = \frac{15}{24} = \frac{5 \times 3}{8 \times 3} = \frac{5}{8}$

Flip the second fraction and multiply.

Multiply the numerators and denominators together

Simplify by Finding common Factors.

c) $\frac{9}{11} \div \frac{5}{3}$

$\frac{9}{11} \div \frac{5}{3} = \frac{9}{11} \times \frac{3}{5} = \frac{9 \times 3}{5 \times 11} = \frac{27}{55}$

Flip the second fraction and multiply.

Multiply the numerators and denominators together

d) $\frac{15}{4} \div \frac{3}{9}$

$\frac{15}{4} \div \frac{3}{9} = \frac{15}{4} \times \frac{9}{3} = \frac{15 \times 9}{4 \times 3} = \frac{135}{12} = \frac{45 \times 3}{4 \times 3} = \frac{45}{4}$

Flip the second fraction and multiply.

Multiply the numerators and denominators together

Simplify by Finding common Factors.

Section D – Higher Only

Worked Example

Calculate $2\sqrt{5} + 2\sqrt{5}$

Step 1: We can only add together surds if they have the same number under the square root. Identify which surds are the same.

Both terms here have 5 under the square root, so they can be added.

Step 2: Look at the number outside the surd. This tells us the multiple of that surd. Use these numbers to add together surds with the same number under the square root.

Both surds have a 2 outside the square root, meaning $2 \times \sqrt{5}$. If we were to write out this calculation in full, it would be $\sqrt{5} + \sqrt{5} + \sqrt{5} + \sqrt{5}$. Collect the terms that are the same.

This gives us $4\sqrt{5}$.

Guided Example

Calculate $5\sqrt{7} - 2\sqrt{7}$

Step 1: We can only add together surds if they have the same number under the square root. Identify which surds are the same.

Both terms have $\sqrt{7}$ in them.

Step 2: Look at the number outside the surd. This tells us the multiple of that surd. Use these numbers to subtract the surds with the same number under the square root.

$5\sqrt{7} - 2\sqrt{7} = 3\sqrt{7}$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

4. Calculate the following, leaving your answer as a surd.

a) $10\sqrt{6} - \sqrt{6}$

$10\sqrt{6} - \sqrt{6} = 9\sqrt{6}$

Both terms have $\sqrt{6}$ in them

b) $2\sqrt{3} + \sqrt{5} + 3\sqrt{3} + 4\sqrt{5}$

$2\sqrt{3} + \sqrt{5} + 3\sqrt{3} + 4\sqrt{5} = (2\sqrt{3} + 3\sqrt{3}) + (\sqrt{5} + 4\sqrt{5}) = 5\sqrt{3} + 5\sqrt{5}$

We have 2 terms each with $\sqrt{3}$ and $\sqrt{5}$ in them, these are like terms.

c) $3\sqrt{6} - 5\sqrt{6}$

$3\sqrt{6} - 5\sqrt{6} = -2\sqrt{6}$

d) $6\sqrt{8} + 3\sqrt{3} - 2\sqrt{8} + 4\sqrt{3}$

$\sqrt{8}$ can be simplified: $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

$6\sqrt{8} + 3\sqrt{3} - 2\sqrt{8} + 4\sqrt{3} = 6(2\sqrt{2}) + 3\sqrt{3} - 2(2\sqrt{2}) + 4\sqrt{3}$

$12\sqrt{2} + 3\sqrt{3} - 4\sqrt{2} + 4\sqrt{3} = 8\sqrt{2} + 7\sqrt{3}$

We have 2 terms each with $\sqrt{3}$ and $\sqrt{2}$ in them, these are like terms.

Section E – Higher Only

Worked Example

Simplify the surd $\sqrt{125}$

Step 1: Find the largest square number that is a factor of the number under the square root.

$$125 = \mathbf{25} \times 5$$

25 *is a square number factor* $(25 = 5^2)$

Step 2: Write the factors under the square root, then split into two surds.

$$\sqrt{125} = \sqrt{25 \times 5} = \sqrt{25} \times \sqrt{5}$$

Step 3: Simplify the surd that is a square number and write the final surd.

$$\sqrt{25} = 5$$

So,

$$\sqrt{125} = \sqrt{25 \times 5} = \sqrt{25} \times \sqrt{5} = 5 \times \sqrt{5} = \mathbf{5\sqrt{5}}$$

Guided Example

Simplify the surd $2\sqrt{18}$

Step 1: Find the largest square number that is a factor of the number under the square root.

Factors of 18: 1×18
3×6
2×9 ← Largest square factor

Step 2: Write the factors under the square root, then split into two surds.

$2\sqrt{18} = 2\sqrt{9 \times 2} = 2(\sqrt{9} \times \sqrt{2})$

Step 3: Simplify the surd that is a square number and write the final surd.

$2(\sqrt{9} \times \sqrt{2}) = 2(3 \times \sqrt{2}) = 6\sqrt{2}$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

5. Simplify the following surds:

a) $\sqrt{72}$

$\sqrt{72} = \sqrt{9 \times 8} = \sqrt{9} \times \sqrt{8} = 3 \times \sqrt{4 \times 2} = 3 \times \sqrt{4} \times \sqrt{2} = 3 \times 2 \times \sqrt{2} = 6\sqrt{2}$

Split the surd into 2 numbers where 1 is a square number & simplify

Split the surd into 2 numbers where 1 is a square number & simplify.

b) $6\sqrt{12}$

$6\sqrt{12} = 6\sqrt{4 \times 3} = 6 \times \sqrt{4} \times \sqrt{3} = 6 \times 2 \times \sqrt{3} = 12\sqrt{3}$

Split the surd into 2 numbers where 1 is a square number & simplify.

c) $5\sqrt{8} + 6\sqrt{28}$

$5\sqrt{8} + 6\sqrt{28} = 5\sqrt{4 \times 2} + 6\sqrt{4 \times 7} = (5 \times \sqrt{4} \times \sqrt{2}) + (6 \times \sqrt{4} \times \sqrt{7})$

Next line: $(5 \times 2 \times \sqrt{2}) + (6 \times 2 \times \sqrt{7}) = 10\sqrt{2} + 12\sqrt{7}$

Split the surd into 2 numbers where 1 is a square number & simplify.

d) $4\sqrt{4} + 6\sqrt{16}$

$4\sqrt{4} + 6\sqrt{16} = 4(2) + 6(4) = 8 + 24 = 32$

Section F – Higher Only

Worked Example

Calculate $4\sqrt{7} \times \sqrt{8}$

Step 1: Check if the surds can be simplified.

$$\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$$

So, the calculation simplifies to: $4\sqrt{7} \times 2\sqrt{2}$

Step 2: When multiplying or dividing surds, perform the operation on the numbers under the square root and the numbers outside separately.

We are now calculating: $4\sqrt{7} \times 2\sqrt{2}$

$$4\sqrt{7} \times 2\sqrt{2} = (4 \times 2)\sqrt{7 \times 2} = 8\sqrt{14}$$

Step 3: Write the final surd, simplifying again if possible.

The final answer is $\mathbf{8\sqrt{14}}$*, which cannot be simplified further.*

Guided Example

Calculate $\frac{4\sqrt{6}}{2\sqrt{12}}$

Step 1: First, check if the surds can be simplified.

$\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$ $\quad \sqrt{6} = \sqrt{3 \times 2} = \sqrt{3} \times \sqrt{2}$

so $\frac{4\sqrt{6}}{2\sqrt{12}} = \frac{4\sqrt{6}}{2 \times 2\sqrt{3}} = \frac{4\sqrt{6}}{4\sqrt{3}} = \frac{4\sqrt{3} \times \sqrt{2}}{4\sqrt{3}}$

Step 2: When multiplying or dividing surds, perform the operation on the numbers under the square root and the numbers outside separately.

$\frac{4\sqrt{3} \times \sqrt{2}}{4\sqrt{3}} = \frac{4}{4} \frac{\sqrt{3}}{\sqrt{3}} \times \sqrt{2} = 1 \times 1 \times \sqrt{2} = \sqrt{2}$

Step 3: Write the final surd, simplifying again if possible.

$\sqrt{2}$ cannot be simplified, answer = $\sqrt{2}$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

6. Calculate the following:

a) $\sqrt{105} \div \sqrt{15}$

$\frac{\sqrt{105}}{\sqrt{15}} = \frac{\sqrt{5\times21}}{\sqrt{5\times3}} = \frac{\sqrt{5}\times\sqrt{21}}{\sqrt{5}\times\sqrt{3}} = \frac{\sqrt{21}}{\sqrt{3}} = \frac{\sqrt{7\times3}}{\sqrt{3}} = \frac{\sqrt{7}\times\sqrt{3}}{\sqrt{3}} = \sqrt{7}$

Split the surd into 2 numbers.

Cancel due to common Factors

Split the surd into 2 numbers.

Cancel due to common Factors

b) $10\sqrt{3} \times 2\sqrt{27}$

$10\sqrt{3}\times2\sqrt{27} = 10\sqrt{3}\times2(\sqrt{9\times3}) = 10\sqrt{3}\times2(\sqrt{9}\times\sqrt{3}) = 10\sqrt{3}\times2(3\times\sqrt{3})$

Split the surd into 2 numbers.

Next line: $10\sqrt{3}\times6\sqrt{3} = 10\times6\times\sqrt{3}\times\sqrt{3} = 60\times3 = 180$

c) $\frac{15\sqrt{10}}{3\sqrt{2}}$

$\frac{15\sqrt{10}}{3\sqrt{2}} = \frac{15(\sqrt{5\times2})}{3\sqrt{2}} = \frac{15(\sqrt{5}\times\sqrt{2})}{3\sqrt{2}} = \frac{15}{3}\times\frac{\sqrt{2}}{\sqrt{2}}\times\sqrt{5} = 5\times1\times\sqrt{5} = 5\sqrt{5}$

Split the surd into 2 numbers.

Cancel due to common Factors

d) $-2\sqrt{12} \times 4\sqrt{12}$

$-2\sqrt{12}\times4\sqrt{12} = -2\times4\times\sqrt{12}\times\sqrt{12} = -8\times12 = -96$

Section G – Higher Only

Worked Example

Rationalise the denominator of the fraction $\frac{5}{\sqrt{5}}$

Step 1: Identify the surd in the denominator.

Looking at the bottom of the fraction, we see the surd present is $\sqrt{5}$.

Step 2: Multiply the numerator and denominator by this surd.

When we multiply the denominator by $\sqrt{5}$, we are squaring a surd, which removes the square root and makes it an integer.

We have to multiply the numerator and denominator by the same number, because this is the same as multiplying it by 1.

$$\frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5}$$

Step 3: Write the final fraction, simplifying if possible.

$$\frac{5\sqrt{5}}{5} = \sqrt{5}$$

Guided Example

Rationalise the denominator of the fraction $\frac{12}{2\sqrt{6}}$

Step 1: Identify the surd in the denominator.

The surd is $\sqrt{6}$

Step 2: Multiply the numerator and denominator by this surd.

$$\frac{12}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{12\sqrt{6}}{2\times 6} = \frac{12\sqrt{6}}{12}$$

Step 3: Write the final fraction, simplifying if possible.

$$\frac{12\sqrt{6}}{12} = \sqrt{6}$$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

7. Rationalise the denominators of the following fractions:

a) $\frac{3}{4\sqrt{7}}$ The surd: $\sqrt{7}$ ← Identify the surd

$$\frac{3}{4\sqrt{7}} = \frac{3}{4\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{4\sqrt{7}\times\sqrt{7}} = \frac{3\sqrt{7}}{4\times 7} = \frac{3\sqrt{7}}{28}$$

b) $\frac{10\sqrt{12}}{3\sqrt{6}}$ The surd in the denominator: $\sqrt{6}$ ← Identify the surd

$$\frac{10\sqrt{12}}{3\sqrt{6}} = \frac{10\sqrt{12}}{3\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{10\sqrt{12\times 6}}{3\times 6} = \frac{10\sqrt{72}}{18} = \frac{10}{18} \times \sqrt{9} \times \sqrt{8} = \frac{5}{9} \times 3 \times 2\sqrt{2}$$

Multiply by '1'

Split the surd into 2 numbers.

Cancel due to common Factors

Next line: $\frac{30\sqrt{2}}{9} = \frac{10\sqrt{2}}{3}$

Cancel due to common Factors

c) $-\frac{4\sqrt{10}}{4\sqrt{16}}$ The surd in the denominator: $\sqrt{16}$

$$-\frac{4\sqrt{10}}{4\sqrt{16}} = -\frac{\sqrt{10}}{\sqrt{16}} = -\frac{\sqrt{10}}{\sqrt{16}} \times \frac{\sqrt{16}}{\sqrt{16}} = -\frac{\sqrt{160}}{16} = -\frac{\sqrt{4\times 4\times 10}}{16} = -\frac{\sqrt{4}\times\sqrt{4}\times\sqrt{10}}{16} = -\frac{2\times 2\times\sqrt{10}}{16}$$

Cancel due to common Factors

Split the surd into 3 numbers.

Next line: $-\frac{4\sqrt{10}}{16} = \frac{4}{16} \times \sqrt{10} = -\frac{\sqrt{10}}{4}$

d) $\frac{9\sqrt{2}}{18\sqrt{3}}$

$$\frac{9\sqrt{2}}{18\sqrt{3}} = \frac{9}{18} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{2}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{2\times 3} = \frac{\sqrt{6}}{6}$$

Cancel due to common Factors